TURBULENT DIFFUSION OF POLYMERS IN A BOUNDARY LAYER

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An account is given in [1] of an experimental study of the diffusion of polymer solutions in a turbulent boundary layer (TBL) at relatively small distances from the slit through which the polymers were introduced into the flow. The results pertain to the so-called intermediate region of TBL diffusion [2], in which the hydrodynamic polymer effects are most strongly manifested.

Our data also embrace the end region of diffusion [2] where the polymer concentration is much lower and the hydrodynamic effects weaker than in the intermediate zone. It was found that the diffusion processes have interesting properties in the end region too. The new data were obtained thanks to improvements in the experimental apparatus and the use of a more accurate method of measuring the local polymer concentration in the TBL.

1. Experimental Conditions. The experiments were carried out in the boundary layer on the flat wall of the working section of a water tunnel measuring 0.36 × 0.15 m in cross section and 5 m in length. The polymer concentration was measured at distances $x = 0.13$-4.5 m and the shear stresses at the wall at distances $x = 0.305$-4.375 m from the line source. The variations of the controlled quantities were as follows: for the flow velocity U at the outer edge of the TBL from 2.5 to 10.0 m/sec, for the Reynolds numbers Re^* (with respect to displacement thickness) from $4 \cdot 10^3$ to $1.1 \cdot 10^5$, for the specific (per unit length of the line source) solution flowrate q from 0.77 to 15.4 cm²/sec, for the initial concentration c_0 of the solution introduced from $2.5 \cdot 10^{-4}$ to $4 \cdot 10^{-3}$ g/cm³. In the experiments we used aqueous solutions of WSR-301 and P-31 polyethylene oxides and R-20-77 (acrylamide-based) copolymer. The line source, transverse to the flow, was located at various points along the working section of the channel at distances $x_S = 0.4$-4.4 m from the effective beginning of the TBL, so that the thickness of the boundary layer at the point of injection of the polymer varied from 0.8 to 5 cm. The dimensions of the outlet of the line source were 0.4 × 130 mm and 0.8 × 130 mm. The inclination of the source to the flow surface resulted in the solution being fed into the flow at an angle $\varphi = 20°$ (in [1] this angle was equal to 7°).

The local polymer concentration in the TBL was measured by taking samples from the flow and then analyzing them by both the polarographic [1] and the newly developed photometric method. The essence of the latter consists in estimating the concentration of admixture from the attenuation of a narrow monochromatic light beam passed through the investigated volume of solution. The polymers were first colored with an alcoholic solution of India ink (technical specification 6-15-458-70). This did not alter the effectiveness of the polymer solutions but made their optical transparency dependent on concentration. A Spectra Physics Model 120 laser served as a source of monochromatic radiation at a wavelength of 0.62 μm. The intensity of the laser beam passing through the investigated volume of solution was registered by a type 55L10 DISA photomultiplier. The sensitivity threshold of the photometric method was about 10^{-7} g/cm³, which is an order better than that of the polarographic method. The photometric method made possible the reliable measurement of small concentrations of polymer even in those regions of the TBL, remote from the line source, in which the Toms effect almost vanishes. The shear stresses were measured by means of inductive transducers with floating elements measuring 20 × 20 mm mounted in the flow surface. The drag measurement error did not exceed ±5%. All the shear stress values realized in the experiments relate to the interval on which the Toms effect is independent of the Reynolds number [3].

2. Results. The simplest characteristic of diffusion transfer in a TBL is the longitudinal diffusion length L, the distance from the line source over which the wall concentration of the diffusing solution decreases by, for example, e times. The more effective the polymer and the greater its flowrate or concentration, the weaker the turbulent mixing in the TBL and the greater the length L. Figure 1 (for notation see Table 1) shows the dependence

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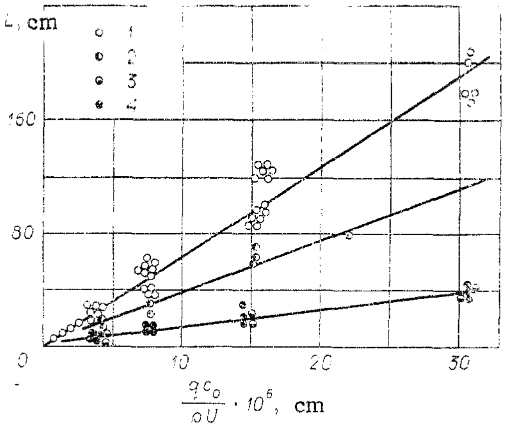

Fig. 1

of L on the flow velocity U, the initial concentration c_0 and the unit flowrate q for various polymers. These relations were obtained at not too large values of qc_0 and correspond to the linear part of the dependence of the length L on qc_0/μ presented in Fig. 3 of [1]. Clearly, L increases in proportion to the complex $qc_0/\rho U$, so that the dimensionless quantity $k = L\rho U/qc_0$ (ρ is the solvent density) can serve as a convenient measure of the ability of a particular polymer to attenuate the turbulent transfer processes. It also takes into account the partial loss of this ability as a result of natural aging of the polymer during prolonged storage or as a result of mechanical degradation of the polymer during use. Of course, in the last analysis, the ability of a polymer to influence turbulent diffusion depends on the molecular structure of the polymer chains, but in hydrodynamic applications measuring the k parameter is a simpler and more reliable procedure than measuring such "fine" physicochemical characteristics as the molecular weight distribution, the conformation parameters of the macromolecules, etc. Below it is shown that k uniquely determines not only the rate of turbulent diffusion of a polymer introduced through a line source into a TBL but also the Toms effect in flows in which the concentration is everwhere constant (e.g., in pipes), where the mean mass diffusion flows are equal to zero.

Table 1 gives the values of the efficiency parameter for the polymers used in the experiments (the figures in brackets following the designation of the polymer indicate the years of "aging" between manufacture and experiment). The last column but one gives values of $k_0 = k/k_{st}$ normalized with respect to the value $k = k_{st} = 6.25\cdot 10^6$ for the polymer WSR-301 (2), which was taken as a standard.

In parallel with the experiments in which we studied diffusion in the TBL, we conducted other experiments in which we determined the ability of the same polymers to reduce the friction drag where there is no mean diffusion transfer. These experiments were carried out on a simple hydraulic apparatus consisting of a 2.8-m length of straight circular tubing 10 mm in inside diameter into which a solution of uniform concentration was introduced from a high-level storage vessel, the flow velocity in the tube being 3 m/sec. The results of the measurements are shown in Fig. 2. The relative decrease in turbulent friction $S_{Tp} = [\tau_w - \tau_p)/\tau_w$ is represented as a function of the effective concentration $k_0 c/\rho$. Here, τ_w and τ_p are the wall friction in water and the polymer solution respectively, and the notation in Fig. 2 (and Fig. 3) is the same as in Fig. 1. It follows from Fig. 2 that with the aid of the parameter k_0 the results for the various polymers and concentrations can be satisfactorily combined in a single dependence

TABLE 1

Designation of polymer	Year of manufacture	$k=L\rho U/qc_0$	k_0	Number assigned to points in Figs. 1-3	Designation of polymer	Year of manufacture	$k=L\rho U/qc_0$	k_0	Number assigned to points in Figs. 1-3
WSR-301 (2)	1976	$6{,}25\cdot 10^6$	1,0	*1*	P-31 (3)	1975	$1{,}25\cdot 10^6$	0,2	*3*
R-20-77 (1)	1977	$3{,}72\cdot 10^6$	0,6	*2*	WSR-301 (9)	1969	$1{,}25\cdot 10^6$	0,2	*4*

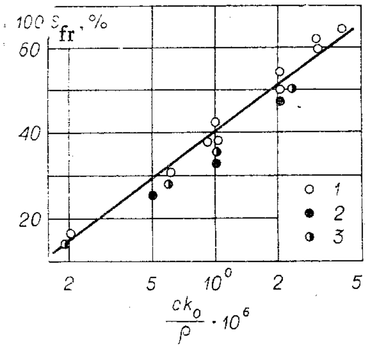

Fig. 2

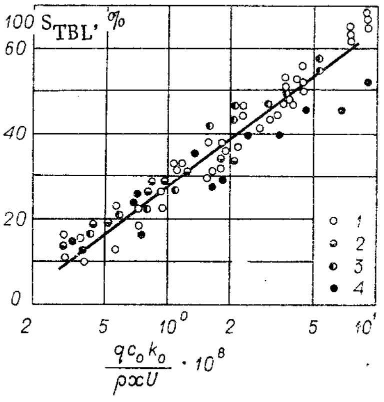

Fig. 3

$$S_{fr} = 0.35\ \lg\ (k_0c/\rho) + 2.49 \tag{2.1}$$

on the interval $k_0c/\rho = 0.2$–4.0.

From Eq. (2.1) there follows

$$k_0 = \frac{\rho}{c}10^{\frac{S_{fr}-2.49}{0.35}}, \tag{2.2}$$

i.e., by means of a simple hydraulic experiment in a tube with a homogeneous solution it is possible to determine the quantity k_0 characterizing the longitudinal diffusion length L in the TBL.

Another method of determining k_0, which is somewhat more complicated but likewise does not involve measuring the local concentration of the polymer diffusing in the TBL, consists in measuring the local friction stress in the boundary layer. From Fig. 3, constructed on the basis of such measurements, there follows

$$S_{TBL} = 0.35\ \ \lg\left(\frac{k_0qc_0}{x\rho U}\right) + 3.08 \tag{2.3}$$

for $(k_0qc_0/x\rho U) \approx 0.4$–$10.0$, whence

$$k_0 = \frac{x\rho U}{qc_0}10^{\frac{S_{TBL}-3.08}{0.35}}, \tag{2.4}$$

where x is the distance downstream from the line source.

It should be noted that the use of Eqs. (2.2) and (2.4) is restricted to the above-indicated intervals of variation of the dimensionless concentrations and flowrates. Extremely large flowrates and concentrations may lead to the "saturation" of the Toms effect [4], when the quantities S_{fr} and S_{TBL} cease to obey laws (2.1) and (2.3), and hence relations (2.2) and (2.4) can no longer be used for determining k_0.

Saturation or near-saturation effects develop in the TBL at relatively small distances from the line source, i.e., in the intermediate diffusion region for which an exponential decrease in the polymer concentration at the wall c_w has been established [1]

$$c_w/c_0 = \exp\left(-\alpha x/L - \beta\right), \tag{2.5}$$

where from the definition of the length L it follows that $\beta = 1 - \alpha$. It was also found (Fig. 4) that the exponent α in (2.5) depends on the angle φ of inclination of the line source to the flow surface and varies from $\alpha = 1.8$ at $\varphi = 20°$ (curve 1) to $\alpha = 0.7$ at $\varphi \leqslant 7°$ (dashed line, from data of [1]). This indicates a slowing of diffusion transfer in the TBL as the angle at which the polymer is fed into the flow decreases, at least on the interval $7° \leqslant \varphi \leqslant 20°$. Figure 4 also gives the results of [5] obtained for $\varphi \sim 0$, which are in good agreement with the data for $\varphi = 7°$. The characteristics of the polymers and polymer solutions

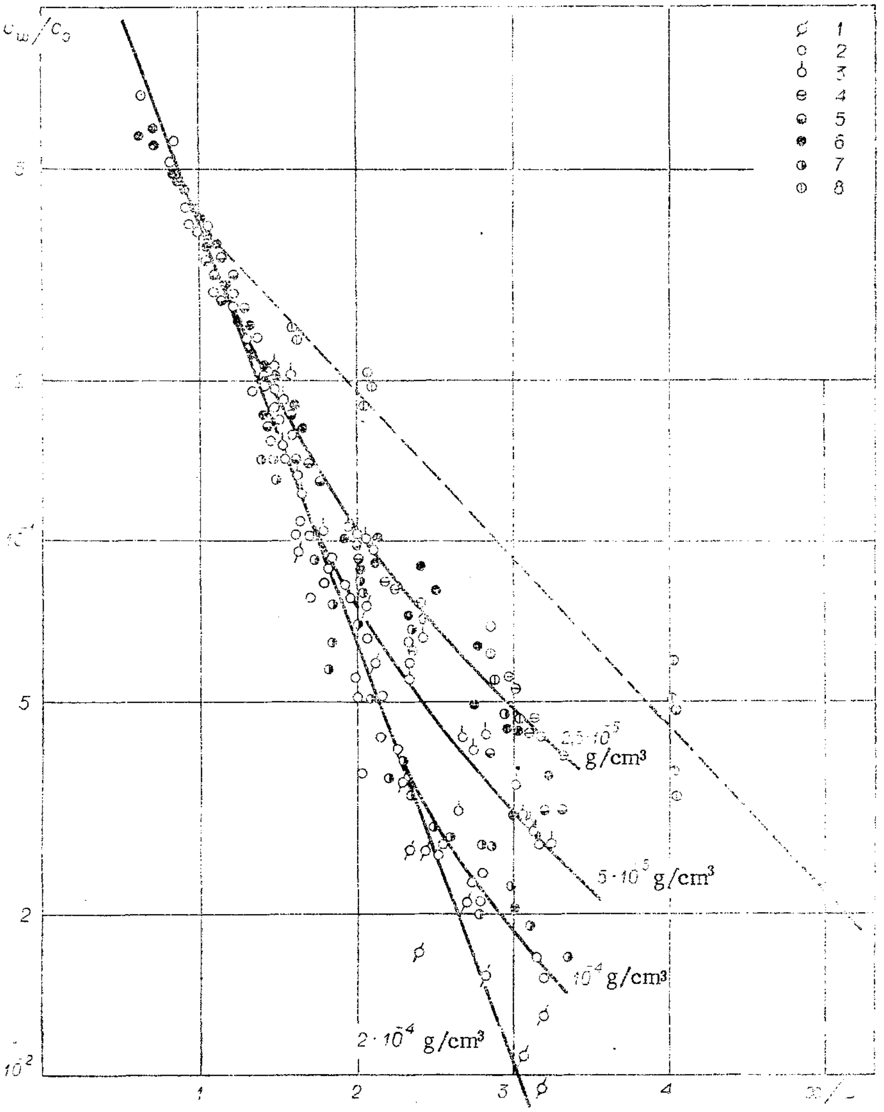

Fig. 4

TABLE 2

Designation of polymer	c_0, g/cm³	k_0	φ, deg	Number assigned to points in Fig. 4	Designation of polymer	c_0, g/cm³	k_0	φ, deg	Number assigned to points in Fig. 4
	$2\cdot10^{-3}$	1,0	20	*1*	P-31 (3)	10^{-3}	0,2	20	*6*
WSR-301 (2)	10^{-3}	1,0	20	*2*	R -20-77 (1)	10^{-3}	0,6	20	*7*
						10^{-4}	1,3	0	*8*
	$5\cdot10^{-4}$	1,0	20	*3*	WSR-301 acc. to data of [5]	$5\cdot10^{-4}$	1,3	0	*8*
	$2{,}5\cdot10^{-4}$	1,0	20	*4*					
WSR-301 (9)	10^{-3}	0,2	20	*5*		10^{-3}	1,3	0	*8*

used to obtain the data in Fig. 4 are summarized in Table 2. It should be noted that at $\varphi \leqslant 7°$ the dependence of the diffusion length L on flow velocity vanishes (see Fig. 3 of [1]), whereas at $\varphi = 20°$ this dependence is clearly observable (see Fig. 1). The experiments showed that varying the location of the line source along the length of the TBL on the interval x_s = 0.4-4.0 m and doubling the width of the source (from 0.4 to 0.8 mm) did not affect the observed diffusion behavior of the polymers.

As may be seen from Fig. 4, the extent of the intermediate region with an exponential dependence (2.5) for the decrease in wall concentration c_w increases with increase in the initial polymer concentration c_0 and the polymer efficiency parameter k_0. Curves 1-4 for the polymer WSR-301 (2) at $c_0 = 2\cdot10^{-3}$, 10^{-3}, $5\cdot10^{-4}$ and $2.5\cdot10^{-4}$ g/cm³, respectively,

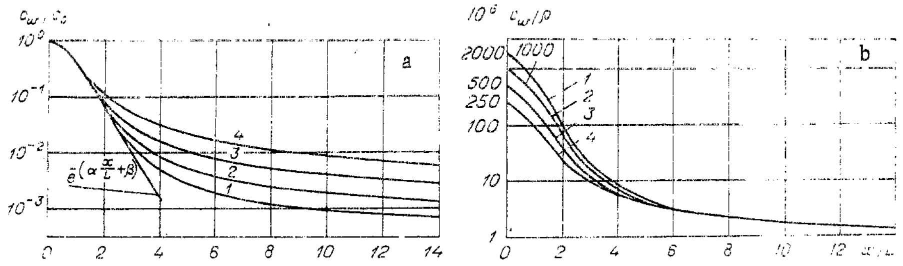

Fig. 5

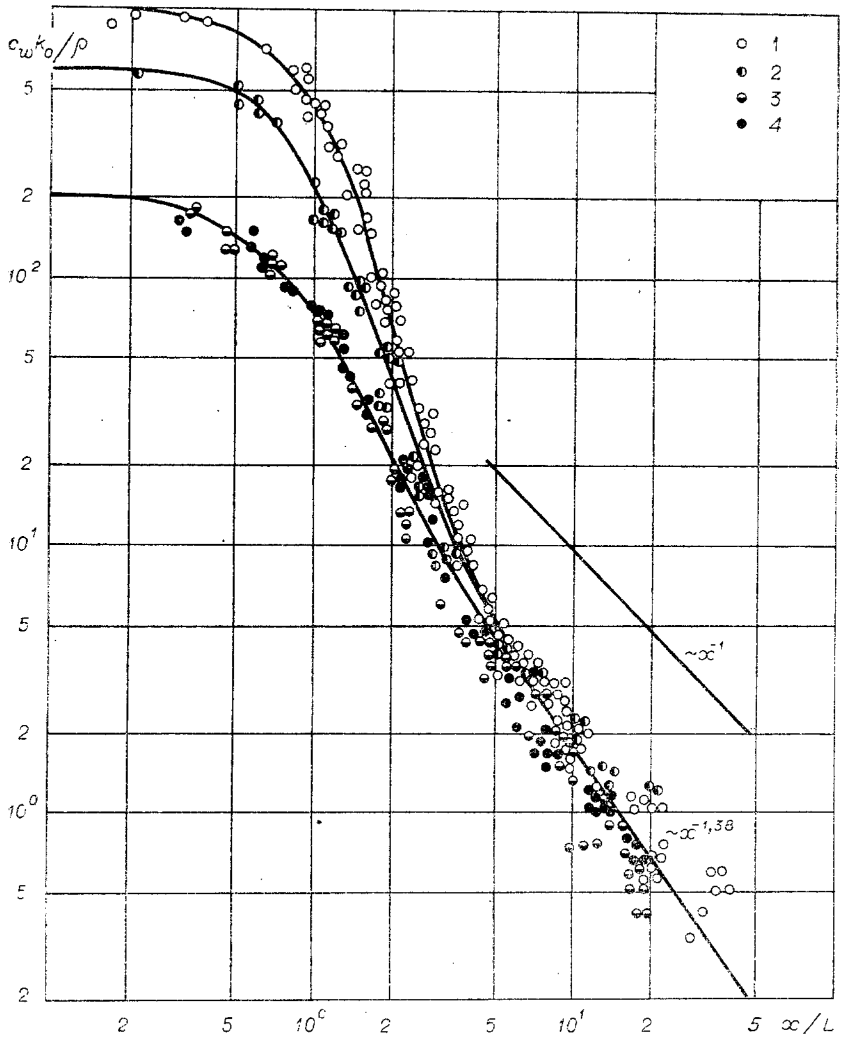

Fig. 6

characterize the onset of the transition from law (2.5) to another diffusion law in the end region. In Fig. 5a, which covers a large interval of dimensionless distances x/L from the line source, the difference between the polymer diffusion laws in the intermediate and end regions is clearly visible. In the end region the dependence $c_W/c_0 = f_1(x/L)$ ceases to be exponential and also universal, in the sense that the function $f_1(x/L)$ describes not a single curve but a family of curves (even for a single polymer of different initial concentration). The results for WSR-301 (2), averaged over numerous measurements, are reproduced in Fig. 5a, b, where curves 1-4 have the same significance as in Fig. 4. If the results of the measurements are represented in the form $c_W/\rho = f_2(x/L)$, then for the same polymer (k_0 = const) they can be reduced to a single dependence (Fig. 5b), but this representation is not universal for different polymers (k_0 = var). Only the form of representation $k_0c_W/\rho = f_3(x/L)$ properly takes into account the differences in the diffusion properties of the different polymers and makes it possible to obtain for large values of x/L a single universal dependence (Fig. 6)

$$\frac{k_0 c_w}{\rho} = 4.16 \cdot 10^{-2} \left(\frac{x}{L}\right)^{-1.38} \qquad (2.6)$$

(the notation in Fig. 6 is taken from Table 1, and for convenience of comparison the data are presented for the same initial concentration c_0, namely 10^{-3} g/cm^3).

With further decrease in the concentration of the polymer in the TBL its influence on the rate of turbulent transfer gradually fades away, and it may therefore be assumed that at x/L > 30-50 relation (2.6) will degenerate into the hyperbolic law $c_w \sim x^{-1}$ characteristic of passive admixtures [2].

On the basis of these experiments we may draw the following conclusions.

In the intermediate diffusion region the polymer concentration decreases along the flow surface in accordance with exponential law (2.5), where the exponent depends on the angle at which the solution is fed into the flow. Relation (2.5) takes into account the effect on diffusion of the initial concentration c_0 and (through the length L) the unit flowrate q of the polymer solution, its effectiveness k_0, and the flow velocity U.

The extent of the region with an exponential concentration decrease increases with the initial concentration, flowrate and effectiveness of the polymer, and vice versa.

In the end region of polymer diffusion in the TBL, the polymer concentration decreases along the wall in accordance with power law (2.6), which does not depend on the initial concentration c_0 but takes into account (through L and k_0) the total flowrate qc_0 and effectiveness of the polymer and the flow velocity U.

The efficiency parameter k_0 and the diffusion length L uniquely associated with it, which are required for it to be possible to use universal diffusion laws (2.5) and (2.6), can be obtained from relation (2.2) on the basis of a simple hydraulic experiment, without having to resort to complex and laborious measurements of the local concentration in the flow.

Hydraulic measurements of the parameter k_0 in tubes also make it possible to calculate the Toms effect in the end (post-exponential, nonsaturating) diffusion region from relation (2.3) for a TBL with a line source.

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